M2 JAN 2010

1) $v = 3t^2 - 4t + 3$

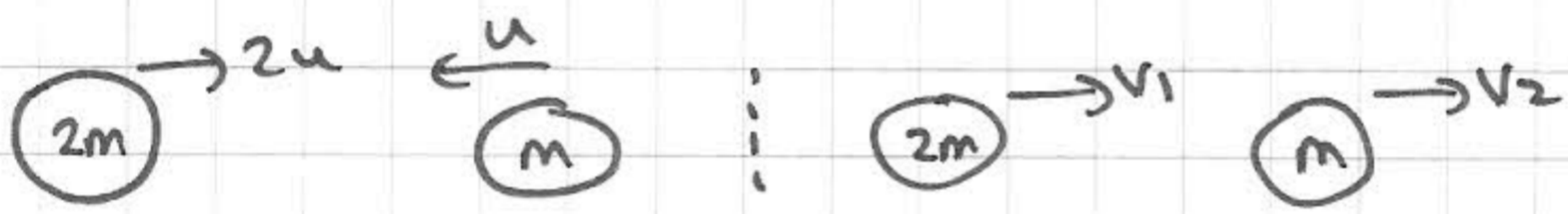
$a = \frac{dv}{dt} = 6t - 4$

$s = \int v\,dt = t^3 - 2t^2 + 3t + c \qquad t=0, s=0 \Rightarrow s = t^3 - 2t^2 + 3t$

at Min Vel $\frac{dv}{dt} = 0 \Rightarrow 6t = 4 \Rightarrow t = \frac{2}{3}$

$s = \left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right) = \frac{8}{27} - \frac{24}{27} + \frac{54}{27} = \frac{38}{27} = \underline{1\frac{11}{27}}$

2)



$2\cancel{m} \times 2u + \cancel{m} \times -u = 2\cancel{m} \times v_1 + \cancel{m} v_2$

$3u = 2v_1 + v_2 \quad \text{—(i)}$

$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{3u} \Rightarrow v_2 - v_1 = 3eu \Rightarrow v_2 = v_1 + 3eu$

into (i)

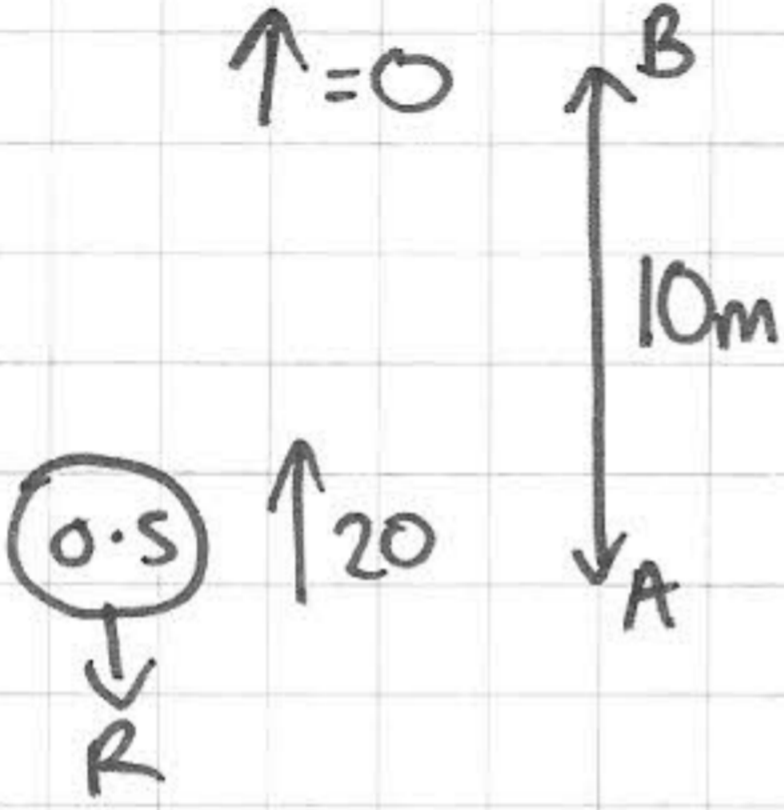
$3u = 2v_1 + (v_1 + 3eu) \Rightarrow \cancel{3}u = \cancel{3}v_1 + \cancel{3}eu$

$v_1 = u - eu \Rightarrow \underline{v_1 = (1-e)u}.$

$v_2 = v_1 + 3eu \Rightarrow v_2 = u - eu + 3eu \Rightarrow v_2 = u + 2eu$

$\underline{v_2 = (1+2e)u}$

3)



$KE_A = \frac{1}{2}(0\cdot 5)(20)^2 = 100$

$PE_B = mgh = \frac{1}{2}g \times 10 = 5g$

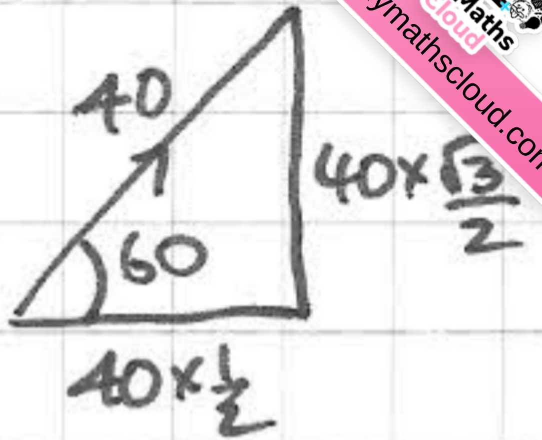
$Wd\ \overrightarrow{AB} = R \times 10 = 10R$

$KE_A = PE_B + Wd$ against R

$100 = 5g + 10R \Rightarrow 10R = 51 \Rightarrow \underline{R = 5\cdot 1\,N}$

4) Mom before $= \frac{1}{4}(30i + 0j) = \frac{15}{2}i$

Mom after $= \frac{1}{4}(20i + 20\sqrt{3}j) = 5i + 5\sqrt{3}j$

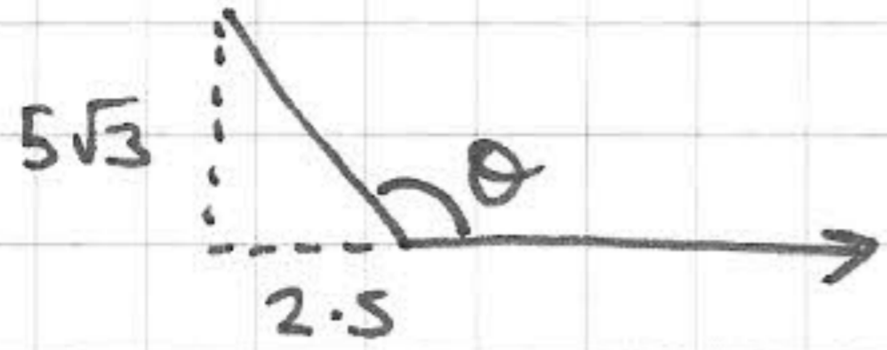


Impulse = Change in Mom $= -\frac{5}{2}i + 5\sqrt{3}j$

Magnitude $= \sqrt{\left(\frac{5}{2}\right)^2 + (5\sqrt{3})^2} = \sqrt{\frac{25}{4} + 75} = \frac{\sqrt{325}}{2} = \frac{5}{2}\sqrt{13}$ Ns

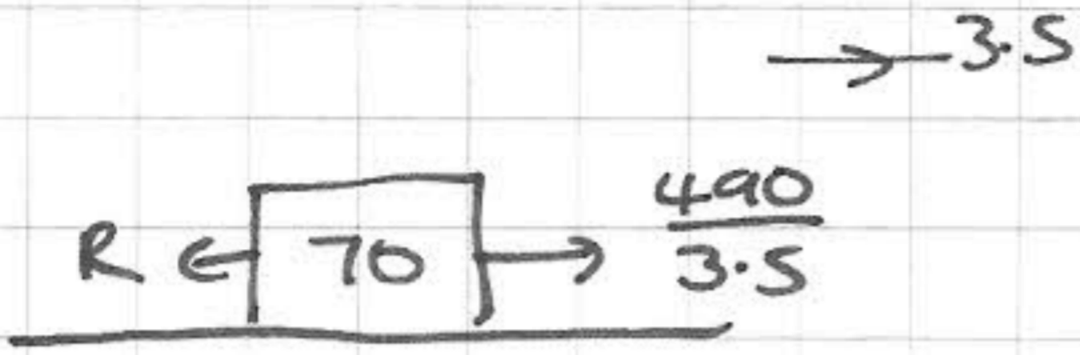
$\tan\theta = -\frac{5}{2}$

(ii)



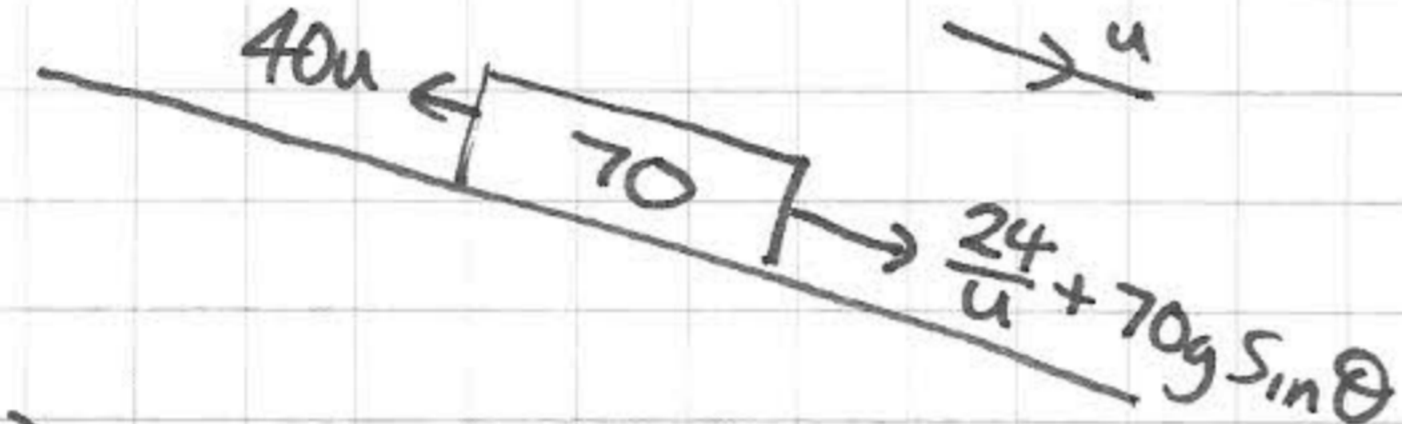
$$\theta = 180 - \tan^{-1}\left(\frac{5\sqrt{3}}{2.5}\right) = 106.1°$$

5)



$$\overrightarrow{Rf} = 0 \Rightarrow \frac{490}{3.5} = R = \underline{140\text{N}}$$

b)



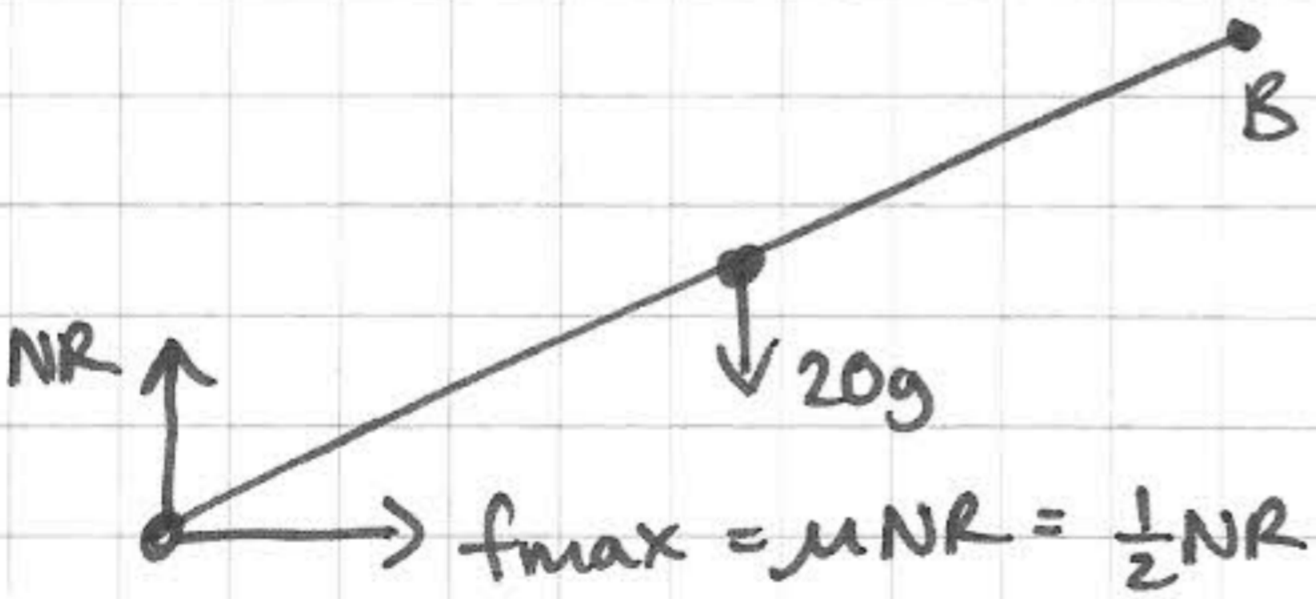
$Rf \searrow = 0$

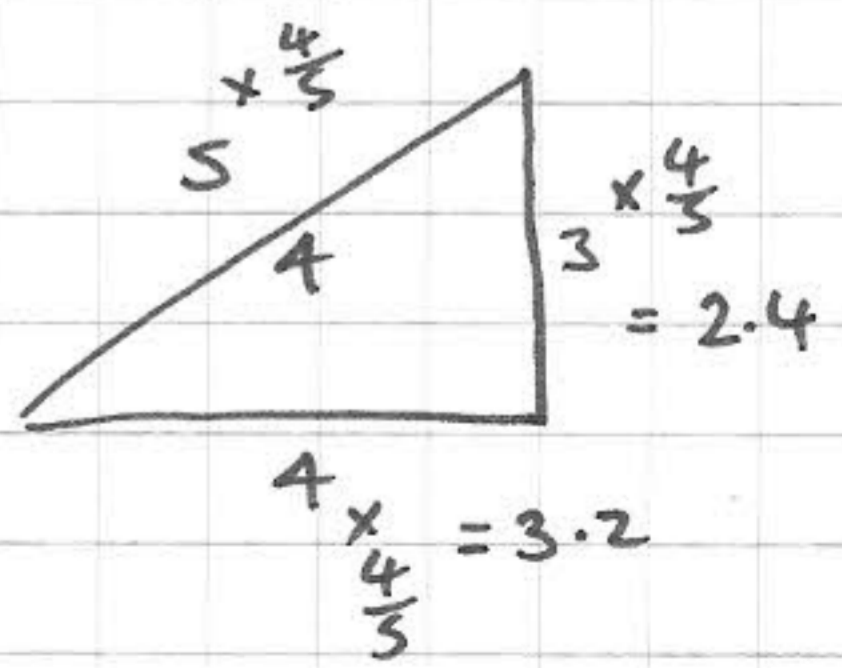
$$\frac{24}{u} + \frac{70g}{14} = 40u \Rightarrow 24 + 5gu = 40u^2$$

$$40u^2 - 49u - 24 = 0$$

$$u = \frac{49 \pm \sqrt{49^2 - 4(40)(-24)}}{80} \Rightarrow u = \frac{49+79}{80} = \underline{1.6\text{ms}^{-1}}$$

6)

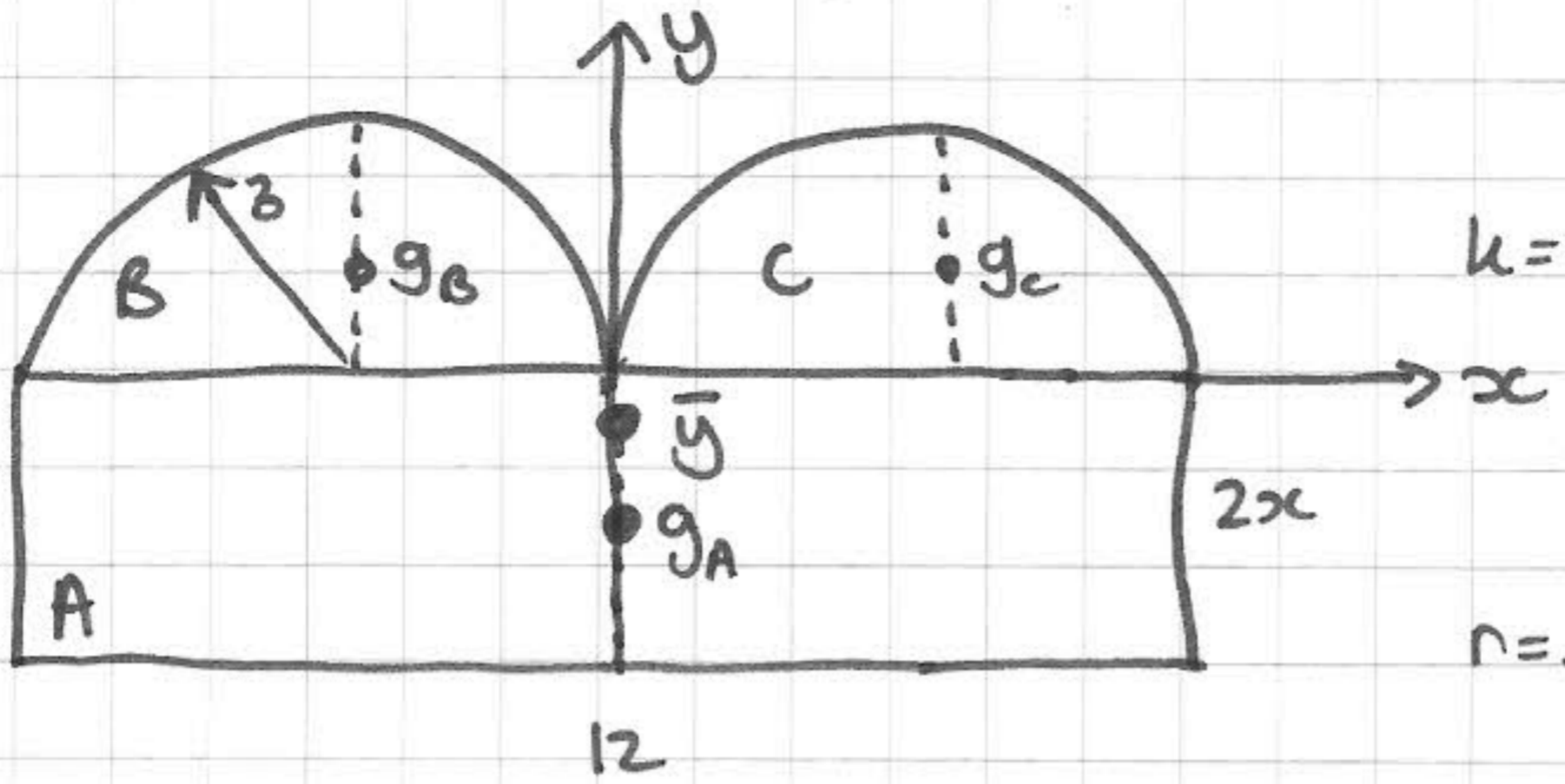




$B \circlearrowright$ $20g \times 1.6 + \frac{1}{2}NR \times 2.4 = NR \times 3.2$

$$32g = 2NR \Rightarrow \underline{NR = 16g\text{ N}}$$

7)



k = mass per unit area

$r=3 \quad \frac{4r}{3\pi} = \frac{12}{3\pi} = \frac{4}{\pi}$

Mass A $= 24x\,k \quad g_A(0, -x)$

Mass B $= \frac{9}{2}\pi\,k \quad g_B(-3, \frac{4}{\pi})$

Mass C $= \frac{9}{2}\pi\,k \quad g_c(3, \frac{4}{\pi})$

Symmetry $\Rightarrow \bar{x} = 0$

Total $= (24x+9\pi)k \quad G(0, -\bar{y})$

x: $24xk\cancel{g} \times -x + \cancel{2}\left(\frac{9}{\cancel{2}}\pi k\cancel{g} \times \frac{4}{\cancel{\pi}}\right) = (24x+9\pi)k\cancel{g} \times -\bar{y}$

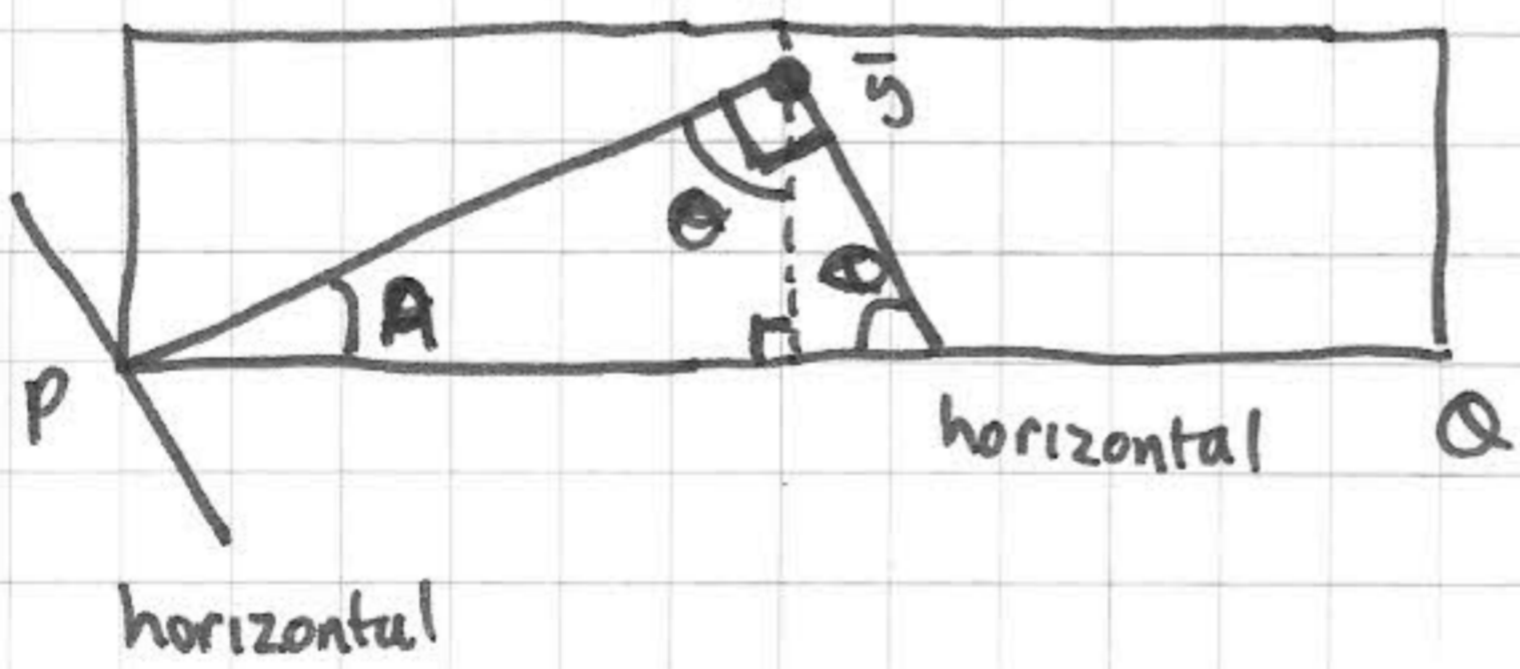
$-24x^2 + 36 = -(24x+9\pi)\bar{y}$

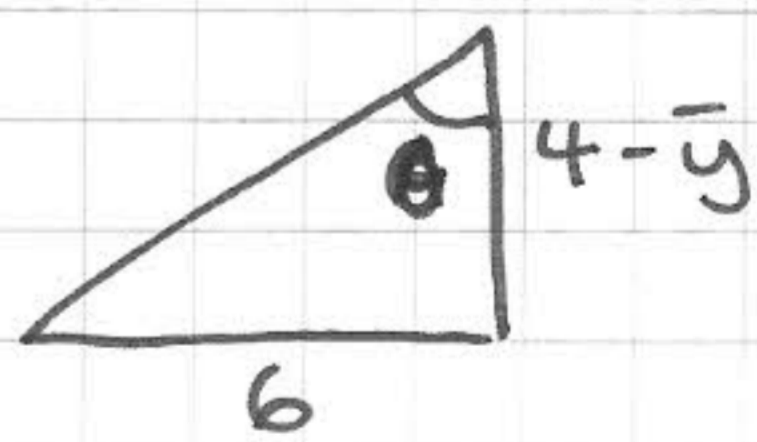
$24x^2 - 36 = (24x+9\pi)\bar{y}$

$\div 3 \quad 8x^2 - 12 = (8x+3\pi)\bar{y}$

$\bar{y} = \frac{4(2x^2-3)}{8x+3\pi}$

b)

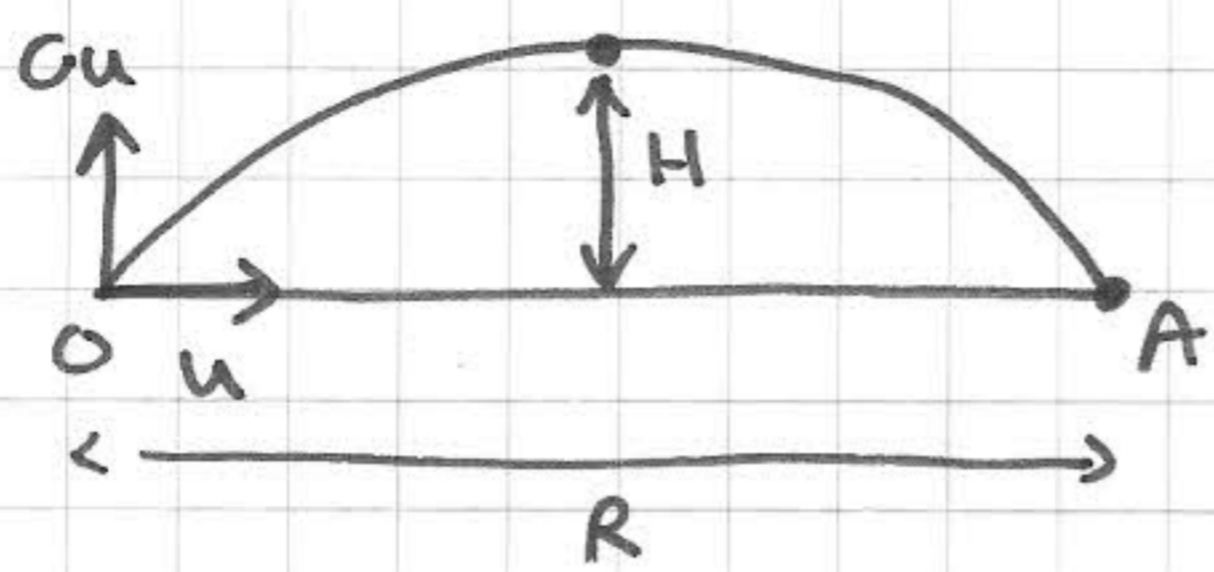




$x=2 \Rightarrow \bar{y} = \frac{4\times5}{16+3\pi} \Rightarrow 4-\bar{y} = \frac{64+12\pi-20}{16+3\pi} = \frac{44+12\pi}{16+3\pi}$

$\tan\theta = \frac{6}{1} \div \frac{44+12\pi}{16+3\pi} \Rightarrow \tan\theta = \frac{6}{1} \times \frac{16\pi+3\pi}{44+12\pi} = \frac{48+9\pi}{22+6\pi}$

8)



$\overrightarrow{H}$ $u = u$ $\quad t = t$ $\quad s = ut$ $\quad x = ut$

$V\uparrow$ $u = cu$ $\quad a = -9.8$ $\quad s = y$

$V\uparrow \Rightarrow s = ut + \frac{1}{2}at^2 \Rightarrow y = cut - 4.9t^2$

from H $\quad t = \frac{x}{u} \qquad \Rightarrow y = c\cancel{u} \times \frac{x}{\cancel{u}} - 4.9\left(\frac{x}{u}\right)^2$

$$y = cx - \frac{4.9x^2}{u^2}$$

b) $u = 7 \Rightarrow x = 7t$

at A $y = 0 \Rightarrow 0 = cx - \frac{4.9x^2}{49} \Rightarrow \frac{1}{10}x^{\cancel{2}} = c\cancel{x}$

$\Rightarrow x = 10c$

$\underline{R = 10c}$

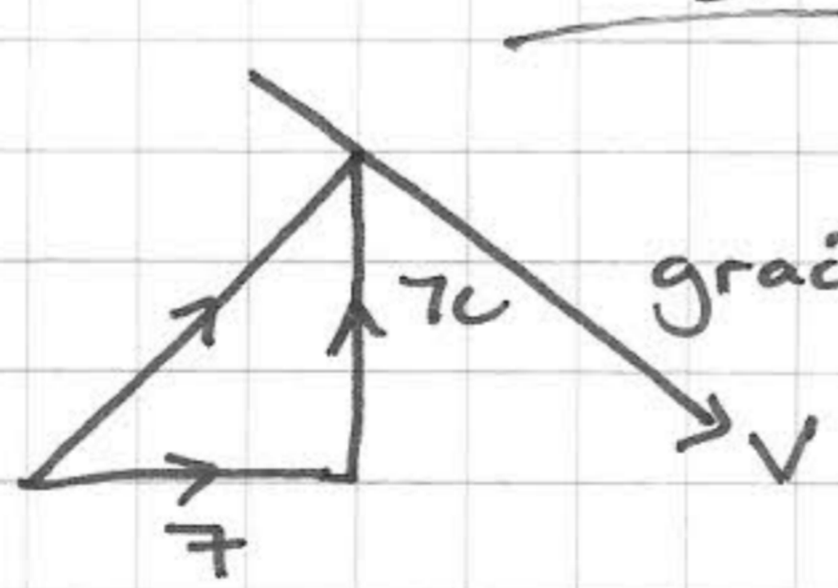
(ii) $7t = 10c \Rightarrow t = \frac{10}{7}c$

So $t = \frac{5}{7}c$ at max height.

$v^2 = u^2 + 2as \Rightarrow 0 = (7c)^2 - 19.6H$

$49c^2 = 19.6H \Rightarrow \underline{H = \frac{5}{2}c^2}$

c) Originally



gradient $= \frac{7c}{7} = c$

at Q Vel is perpendicular $\Rightarrow$ gradient $= -\frac{1}{c}$

$7 \rightarrow$ doesn't change!

$\frac{y}{7} = -\frac{1}{c} \Rightarrow y = -\frac{7}{c}$

$u\uparrow = 7c \quad v\uparrow = -\frac{7}{c}$

$a = -9.8$

$v = u + at \quad -\frac{7}{c} = 7c - 9.8t$

$9.8t = 7c + \frac{7}{c} = \frac{7c^2 + 7}{c} \Rightarrow t = \frac{7c^2 + 7}{9.8c}$

$\overrightarrow{H} \quad x = 7t \quad x = \frac{7(7c^2+7)}{9.8c} = \frac{5(7c^2+7)}{7c} = \frac{5(c^2+1)}{c}$